This question paper contains 7 printed pages]

Your Roll No. ....

8864

B.Sc. (Hons.) Computer Science/V Sem. C

Paper CS—504 : Numerical Analysis and Scientific Computing

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(*Write your Roll No. on the top immediately on receipt of this question paper.*)

All questions of Section A are compulsory.

In Section B, attempt any *four* questions.

Use of non-programmable scientific calculator is allowed. Symbols have their usual meaning.

Section A 7×5=35

(Compulsory)

1. Determine the smallest interval in which the result using true instead of rounded values must be located for the following :

(*a*) 1.1062 + .947

(*b*) (2.747) (6.83)

P.T.O.

2. Assume $f(x)$ is twice continuously differentiable on : $[a, b]$, $f(a) < 0$, $f(b) > 0$ and $f'(x) \geq 0$, $f''(x) > 0$ for $a \leq x \leq b$. Then prove that the iterates x_n are strictly decreasing to α, and iterates z_n are strictly increasing to α, where α is the root of $f(x) = 0$.

3. Assume α is a root of $x = g(x)$ and that $g(x)$ is p times continuously differentiable for all x near to α, for some $p \geq 2$ furthermore, assume $g'(\alpha) = \ldots\ldots = g^{(p-1)}(\alpha) = 0$. Then prove that if the initial guess x_0 is chosen sufficiently close to α, the iteration :

$$x_{n+1} = g(x_n),\ n \geq 0$$

will have the order of convergence p, and :

$$\lim_{x \to \infty} \frac{\alpha - x_{n+1}}{(\alpha - x_n)^p} = (-1)^{p-1} \frac{g^{(p)}(\alpha)}{p!}$$

4. If $f(x) = \dfrac{1}{x}$, then evaluate Newton divided difference $f[a, b, c, d]$.

5. Evaluate :

$$I = \int_0^1 \frac{dx}{1 + x^2}$$

using Simpson's rule.

6. Let :

$$A = \begin{bmatrix} 5 & -5 & 7 \\ -4 & 2 & -4 \\ -7 & -4 & 5 \end{bmatrix}.$$

Calculate :

(*i*) $\|A\|_f$,

(*ii*) $\|A\|_\infty$, and

(*iii*) $\|A\|_1$.

7. Using Adams-Moulton method, find an approximate value of y, when $x = 0.4$, given that :

$$\frac{dy}{dx} = -2x - y \,;$$

$y(0) = -1,$

$y(0.1) = -0.9145,$

$y(0.2) = -0.8561,$

$y(0.3) = -0.8224.$

Section B

4×10=40

Attempt any *four* :

8. (*a*) Consider Newton-Raphson Method for finding the positive square root of $a > 0$. Derive the following results assuming :

$$x_0 > 0,\ x_0 \neq \sqrt{a}$$

(*i*) $x_{n+1}^2 - a = \left[\frac{x_n^2 - a}{2x_n}\right]^2,\ n \geq 0$

and thus $x_n > \sqrt{a}$ for all $n > 0$.

(*ii*) The iterates $\{x_n\}$ are strictly decreasing sequence for $n \geq 1$.

(*b*) Show that :

$$x_{n+1} = \frac{x_n(x_n^2 + 3a)}{3x_n^2 + a},\ n \geq 0$$

is a third order method for computing $\sqrt{a}$.

Calculate :

$$\lim_{n\to\infty} \frac{\sqrt{a} - x_{n+1}}{(\sqrt{a} - x_n)^3}$$

assuming x_0 has been chosen sufficiently close to α.

9. (*a*) Let $x_0, x_1, \ldots, x_n$ be distinct real numbers, and let f be a given function with $(n+1)$ continuous derivatives on the interval :

$$I_t = H\{t, x_0, \ldots\ldots, x_n\}$$

with t some given real numbers. Then prove that there exists $\xi \in I_t$ with :

$$f(t) - \sum_{J=0}^{n} f(x_J)\, l_J(t) = \frac{(t - x_0) \ldots (t - x_n) \, . \, f^{n+1}(\xi)}{(n+1)!}$$

where :

$$H\{t, x_0 \ldots, x_n\}$$

denotes the smallest internal containing all of the real numbers $t, x_0, \ldots, x_n$.

(*b*) For $k \geq 0$, prove that :

$$f[x_0, x_1 \ldots\ldots, x_k] = \frac{1}{k!\,.\,h^k} \Delta^k f_0$$

10. (*a*) Determine the step-size h that can be used in the tabulation of $f(x) = \sin x$ in the interval [1, 3] so that the linear interpolation will be correct to four decimal places after rounding.

(*b*) Obtain a linear polynomial approximate to the function $f(x) = \ln(x)$ on the [1, 2] using the least square approximation.

11. (*a*) Obtain the error formula for composite trapezoidal rule.

(*b*) Approximate the following integration using the Gaussian quadrature for $\eta = 2$.

$$\int_0^{\pi} e^x \cos x \, . \, dx$$

12. (*a*) Solve the following system of equations using Jacobi method :

$$\begin{aligned} 4x + y + z &= 2 \\ x + 5y + 2z &= -6 \\ x + 2y + 3z &= -4 \end{aligned}$$

(perform 3 iterations only)

Solve the following differential equation using fourth order Runge-Kutta method :

$$\frac{dy}{dx} = 1 + \frac{y}{x}$$

ven $y(1) = 1$

$x = 1.5$, taking $h = 0.5$

ite a short note on Rayleigh-Ritz method.

te the codes in MATLAB/MATHEMATICA/MAPLE solve the following integral using Trapezoidal :

$$\int_0^1 \frac{dx}{1+x}$$